(one-fifth to one-tenth of an instrumented model), and the ability to obtain isotherms easily. Significant improvements in accuracy could be obtained by using a sequence camera instead of a movie camera and by painting a coordinate network on the models.

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Separation of Satellites in Near-Circular Orbits by Circumferential Impulse

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Nomenclature

a = semimajor axis
b = semiminor axis
e = orbital eccentricity
E = eccentric anomaly
F = principal focus of ellipse
M = mean anomaly
n = number of orbits subsequent to separation
0 = origin of rectangular coordinate system
$P(x, y)$ = position at time t of satellite in lower energy orbit
P' = position at time t of satellite in higher energy orbit
r = central radius
S = separation distance
t = time from separation
T = orbit period
V = satellite velocity
ΔV = impulsive velocity increment
β = view angle (Fig. 2)
η = true anomaly
θ = elevation angle (Fig. 2)

Superscript

* = condition at passage point

A SATELLITE in a central gravitational field is in near-circular orbit. A second satellite is separated from the first by means of a small impulsive velocity increment directed collinearly or anticollinearly with the velocity vector. It is desired to describe the flight-path histories following separation in terms of distance and view angle between satellites.

By the method of differentials, it is found that, to the first order, the lower-energy satellite overtakes that of higher energy when both have traveled through 73.09° of central angle η (Fig. 1). The separation distance S at a given η is related linearly to the central radius r and the nondimensional separation velocity $\Delta V/V$, but the view angle β (the angle between the local horizontal, for the lower-energy satellite and the line connecting the two satellites), is independent of r and $\Delta V/V$.

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Analysis

The time from periapsis in an elliptical orbit can be expressed in terms of orbit period T and mean anomaly M

$$t = TM/2\pi \tag{1}$$

Differentiating,

$$dt = (TdM + MdT)/2\pi \qquad M = E - e\sin E \tag{2}$$

where E is the eccentric anomaly (Fig. 1), and e is the orbital eccentricity; to the first order, $dM = dE - de\sin E$, so that

$$dt = [T(dE - de\sin E) + E\,dt]/2\pi \tag{3}$$

Since comparison of elements is to be made at a given time, $dt = 0$; it follows that

$$dE = de\sin E - E\,dt/T \tag{4}$$

The lower-energy satellite initially trails its sister, but by losing altitude it gains velocity and overtakes its sister during the first orbit; a second passage does not occur until many orbits later. At passage, the two satellites have equivalent central angle η, measured about the principal focus from the initial apsidal point (Fig. 1). The central angle thus defined is equivalent to the true anomaly when measured from periapsis, or 180° minus the true anomaly when measured from apoapsis. In the present context, it is mathematically permissible to assume that the problem always initiates at periapsis, η then being equivalent to the true anomaly. The relationship between E and η is known to be [Ref. 1, Eq. (4-113)]

$$\tan(\eta/2) = [(1+e)/(1-e)]^{1/2}\tan(E/2) \approx (1+e)\tan(E/2) \tag{5}$$

Differentiation of (5) gives

$$\tfrac{1}{2}\sec^2(\eta/2)d\eta = \tfrac{1}{2}\sec^2(E/2)dE + de\tan(E/2)$$

But, at the point of passage, $d\eta = 0$, so that

$$dE^* = -[2de\tan(E^*/2)]/[\sec^2(E^*/2)] = -de\sin E^* \tag{6}$$

where the asterisk indicates the initial passage point. Substituting (6) into (4) and solving for E^* gives $E^* = (\frac{4}{3})\sin E^* = 73.09°$. This result confirms the limit case indicated by Milstead [Ref. 2, Eq. (7)]. The separation distance S^* is found next. From Ref. 1, Eq. (4-99),

$$r = a(1 - e\cos E) \tag{7}$$

where a is the semimajor axis of the ellipse. Differentiating this relation,

$$dr/a = (da/a) - de\cos E \approx dr/r \tag{8}$$

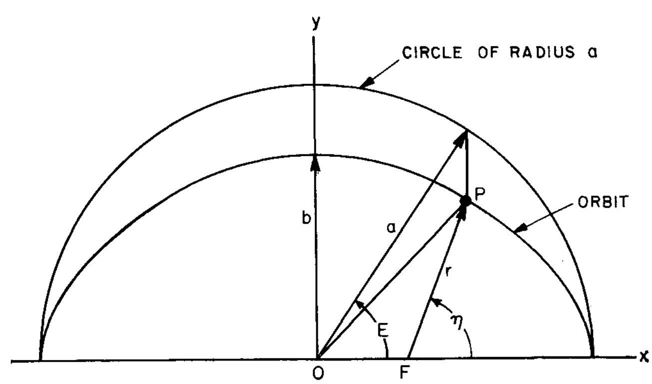

Fig. 1 Orbit notation.

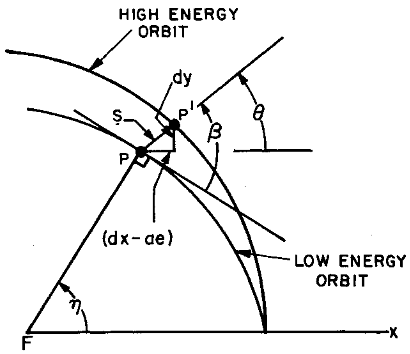

Fig. 2 View angle geometry.

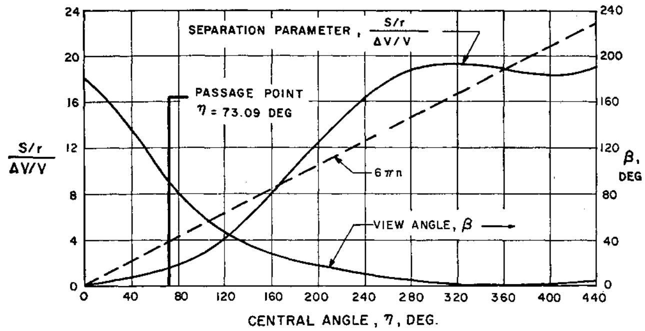

Fig. 3 Separation parameter and view angle vs central angle.

For the present family of near-circular orbits, $de = da/a$, so that (8) becomes, at passage,

$$S^*/r = de(1 - \cos E^*) \tag{9}$$

Furthermore, for near-circular orbits, $de = 2\Delta V/V$, so that (9) becomes

$$S^*/r = 1.418(\Delta V/V) \tag{10}$$

For a 500-naut-mile-alt earth orbit, for instance, (10) indicates that $S^* = 1410$ ft/ft/sec. To obtain a more general result for separation distance, the vector sum of dr and $rd\eta$ could be sought; however, it is also convenient to establish a rectangular coordinate system. This will provide a check of the previous development. In Fig. 1, the x, y system with origin 0 at the center of the ellipse is chosen. Note that this is a noninertial system, and the origin translates along x such that $OF = ae$. At a point $P(x, y)$ on the ellipse,

$$x = a \cos E \tag{11}$$

and

$$y = b \sin E = a(1 - e^2)^{1/2} \sin E \approx a \sin E \tag{12}$$

to the accuracy of this analysis. Differentiating (11) and (12),

$$dx/a = -\sin E\, dE + (da/a) \cos E \tag{13}$$

$$dy/a = \cos E + (da/a) \sin E \tag{14}$$

The separation distance can be expressed (Fig. 2)

$$S/r \approx S/a = \{[(dx/a) - de]^2 + (dy/a)^2\}^{1/2} \tag{15}$$

recalling the origin translation. Substituting (13) and (14) in (15),

$$S/r = [dE^2 + 2de \sin E\, dE + 2de^2(1 - \cos E)]^{1/2} \tag{16}$$

Substituting for dE from (6) for the passage point gives the result

$$S^*/r = de(1 - \cos E^*)$$

which is identical to (9). For near-circular orbits, $dT/T = (\frac{3}{2})de$, so that Eq. (4) can be written

$$dE = de(\sin E - \tfrac{3}{2}E) = (2\Delta V/V)(\sin E - \tfrac{3}{2}E) \tag{17}$$

Substitution of (17) into (16) gives the desired result

$$(S/r)/(\Delta V/V) = 2[3 \sin^2 E - 6E \sin E + \tfrac{9}{4}E^2 + 2(1 - \cos E)]^{1/2} \tag{18}$$

wherein, to the accuracy of Eq. (5), $E = \eta$.

Results

In Fig. 3, the separation parameter $(S/r)/(\Delta V/V)$ is shown as a function of η for somewhat more than one orbit; for greater η, the function will continue to oscillate about the gradient $6\pi n$, where n is the number of orbits.

In some cases, the angular relationship of the two satellites is of interest (e.g., when a region of sensor interference is to be determined). In Fig. 2, a "view angle" β is defined as the angle between the local horizontal for the lower-energy satellite and the line connecting the two satellites, such that

$$\beta = (90 - \eta) + \theta$$

$$\theta = \tan^{-1}\{(dy/a)/[(dx/a) - e]\} \tag{19}$$

Or, employing (13, 14, and 17),

$$\theta = \tan^{-1}\left[\frac{\cos E\,(\sin E - 3E/2) + \sin E}{(\cos E - 1) - \sin E(\sin E - 3E/2)}\right] \tag{20}$$

Again assuming that $E = \eta$, the view angle β is plotted in Fig. 3. Interestingly enough, β is independent of the orbital altitude or separation velocity.

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